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SPECIAL CASE OF A TRAVELING WAVE IN ONE MODEL OF A MULTICOMPONENT MEDIUM

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The Kuropatenko model for a multicomponent medium whose components are polytropic gases is considered. It is assumed that, as $x \to \pm\infty$, the multicomponent medium is in a homogeneous state with constant gas-dynamic parameters — velocity, pressure, and temperature. For the traveling wave flows, conditions similar to the Hugoniot conditions are obtained and used to uniquely determine the flow parameters for $x \to -\infty$ from the flow parameters $x \to +\infty$ and traveling wave velocity.

Key words: *multicomponent medium, traveling wave, Hugoniot conditions.*

Kuropatenko [1] proposed a mathematical model for describing flows of multicomponent media based on quasilinear equations with partial derivatives constructed from the conservation laws for a mixture obtained from the conservation laws for the components. The model takes into account both the pair interaction of the components and the cluster interaction of the components with an introduced virtual continuous medium. An advantage of this model is that it is closed: the system contains an identical number of equations and required functions and its closure does not require additional hypotheses.

The present paper considers the case where each of the N components ($N \geq 2$) of the model of a multicomponent medium proposed in [1] are an ideal polytropic gas with equations of state of the form

$$P_i = (\gamma_i - 1)c_{vi}^0 \rho_i T_i, \qquad E_i = c_{vi}^0 T_i, \qquad i = 1, 2, \ldots, N.$$

Here P_i, ρ_i, T_i, and E_i are the pressure, density, temperature, and internal energy of the ith component, respectively, and $\gamma_i = \text{const} > 1$ and $c_{vi}^0 = \text{const} > 0$ are the polytropic exponent and the specific heat capacity of the ith component.

For each component, the squared sound velocity c_i is equal to

$$c_i^2 = \gamma_i(\gamma_i - 1)c_{vi}^0 T_i.$$

In addition to the indicated functions, multicomponent flows are characterized by the volume concentrations α_i and partial densities of the components $\sigma_i = \alpha_i \rho_i$.

We consider the case of plane symmetric flows

$$\frac{\partial}{\partial x_2} = \frac{\partial}{\partial x_3} = 0,$$

where the velocity of each component is parallel to the Ox axis ($x = x_1$; x_1, x_2, and x_3 are Cartesian coordinates in physical space), i.e., it is given by the scalar quantity u_i.

Thus, the required functions are the $4N$ functions σ_i, u_i, T_i, and α_i ($1 \leq i \leq N$) which depend on the time t and the coordinate x and are solutions of the system of $4N$ equations [1] written in dimensionless variables:

$$\frac{\partial \sigma_i}{\partial t} + \frac{\partial\,(\sigma_i u_i)}{\partial x} = 0,$$

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$$
\begin{gathered}
\frac{\partial\,(\sigma_i u_i)}{\partial t} + \frac{\partial\,(\sigma_i u_i^2)}{\partial x} + \frac{\partial p_i}{\partial x} + \frac{\partial f_{ksi}}{\partial x} - r_{si} = 0, \\
\frac{\partial\,(\sigma_i \varepsilon_i)}{\partial t} + \frac{\partial\,[(p_i + \sigma_i \varepsilon_i) u_i]}{\partial x} + \frac{\partial\,(f_{ksi} u_i)}{\partial x} + \frac{\partial q_{si}}{\partial x} - \varphi_{si} - A_{si} = 0, \\
p_i \Big(\frac{\partial \alpha_i}{\partial t} + u_i \, \frac{\partial \alpha_i}{\partial x} \Big) + \alpha_i (u - u_i)\, \frac{\partial p_i}{\partial x} - 2\alpha_i \, \frac{\partial q_{si}}{\partial x} = 0.
\end{gathered} \tag{1}
$$

Here

$$
p_i = \alpha_i P_i \equiv (\gamma_i - 1) c_{vi}^0 \sigma_i T_i, \qquad \varepsilon_i = E_i + 0.5 u_i^2
$$

are the partial pressure and specific total energy of the ith component, respectively;

$$
f_{ksi} = -\frac{1}{2}\, \sigma_i (u - u_i)^2, \qquad r_{si} = \alpha_i \sum_{j=1}^{N} \alpha_j a_{ji} (u_j - u_i), \qquad q_{si} = \frac{1}{2}\, (p_i + \sigma_i E_i)(u - u_i),
$$

$$
\varphi_{si} = \alpha_i \sum_{j=1}^{N} \alpha_j [b_{ji}(P_j - P_i) + c_{ji}(T_j - T_i)], \qquad A_{si} = 0.5\alpha_i \sum_{j=1}^{N} \alpha_j a_{ji} (u_j^2 - u_i^2),
$$

$$
u = \frac{1}{\sigma} \sum_{i=1}^{N} \sigma_i u_i, \qquad \sigma = \sum_{i=1}^{N} \sigma_i
$$

are the virtual velocity and partial density, respectively;

$$
a_{ij} = a_{ji} > 0, \quad b_{ij} = b_{ji} > 0, \quad c_{ij} = c_{ji} > 0 \qquad (1 \le i \leqslant N, \quad 1 \le j \le N)
$$

are constant exchange coefficients, and $a_{ii} = b_{ii} = c_{ii} = 0$.

Remark 1. In the derivation of the model of the multicomponent medium in [1], the equality

$$
\sum_{i=1}^{N} \alpha_i = 1 \tag{2}
$$

is postulated without being directly attached to system (1); therefore, for each constructed solution of system (1), it is necessary to additionally verify the validity of equality (2). In [2], this is performed for flows in equilibrium in velocities. In the present paper, concrete solutions of system (1) are not constructed; therefore, there is no need to verify the validity of equality (2).

Next, we write the quantities p_i, f_{ksi}, r_{si}, ε_i, q_{si}, φ_{si}, and A_{si} from system (1) in terms of the required functions σ_i, u_i, T_i, and α_i, and, to study the properties of the partial solutions of system (1) — traveling waves — we introduce the independent variables τ and z:

$$
\tau = t, \qquad z = x - Dt, \qquad D = \text{const}
$$

(D is the traveling wave velocity). Since a traveling wave is a function of only z, we set $\partial/\partial\tau = 0$. Then, system (1) becomes the following system of ordinary differential equations:

$$
\begin{gathered}
-D\, \frac{\partial \sigma_i}{\partial z} + \frac{\partial\,(\sigma_i u_i)}{\partial z} = 0, \\
-D\, \frac{\partial\,(\sigma_i u_i)}{\partial z} + \frac{\partial\,(\sigma_i u_i^2)}{\partial z} + \frac{\partial\,[(\gamma_i - 1) c_{vi}^0 \sigma_i T_i]}{\partial z} - \frac{1}{2}\, \frac{\partial\,[\sigma_i (u - u_i)^2]}{\partial z} - \alpha_i \sum_{j=1}^{N} \alpha_j a_{ji} (u_j - u_i) = 0, \\
-D\, \frac{\partial\,(c_{vi}^0 \sigma_i T_i + 0.5 \sigma_i u_i^2)}{\partial z} + \frac{\partial\,[(\gamma_i c_{vi}^0 \sigma_i T_i + 0.5 \sigma_i u_i^2) u_i]}{\partial z} - \frac{1}{2}\, \frac{\partial\,[\sigma_i (u - u_i)^2 u_i]}{\partial z} \\
+ \frac{1}{2}\, \frac{\partial\,[\gamma_i c_{vi}^0 \sigma_i T_i (u - u_i)]}{\partial z} - \alpha_i \sum_{j=1}^{N} \alpha_j \Big(b_{ji}(P_j - P_i) + c_{ji}(T_j - T_i) + \frac{1}{2}\, a_{ji} (u_j^2 - u_i^2) \Big) = 0;
\end{gathered} \tag{3}
$$

$$-D(\gamma_i-1)c_{vi}^0\sigma_i T_i\,\frac{\partial\alpha_i}{\partial z}+c_{vi}^0\alpha_i(u-u_i)T_i\Big(\gamma_i\,\frac{\sigma_i}{\sigma}-1\Big)\frac{\partial\sigma_i}{\partial z}$$

$$+\,\gamma_i c_{vi}^0\alpha_i\sigma_i T_i\Big(1-\frac{\sigma_i}{\sigma}\Big)\frac{\partial u_i}{\partial z}-c_{vi}^0\alpha_i(u-u_i)\sigma_i\,\frac{\partial T_i}{\partial z}+(\gamma_i-1)c_{vi}^0\sigma_i T_i u_i\,\frac{\partial\alpha_i}{\partial z}$$

$$+\,\gamma_i c_{vi}^0\,\frac{\alpha_i\sigma_i T_i}{\sigma}\sum_{j=1,j\neq i}^{N}(u-u_j)\,\frac{\partial\sigma_j}{\partial z}-\gamma_i c_{vi}^0\,\frac{\alpha_i\sigma_i T_i}{\sigma}\sum_{j=1,j\neq i}^{N}\sigma_j\,\frac{\partial u_j}{\partial z}=0. \qquad (4)$$

Remark 2. Since system (1) is invariant under the Galilean transformation [1], the solutions of system (3), (4) are in fact steady-state solutions of system (1) in the corresponding system of coordinates.

For a one-component medium in gas dynamics, the functional equations linking the shock wave (SW) velocity D and the gas parameters $\boldsymbol{U}^0$ and $\boldsymbol{U}^1$ on the different sides of the SW are called the Hugoniot conditions [3, 4]. These conditions are derived using the following method [4]: in the gas-dynamic equations describing one-component flows, the above transformation to the variables τ and z is made and it is assumed that $\partial/\partial\tau=0$, i.e., partial solutions of the equations of gas dynamics — traveling waves — are considered. The result is a system of ordinary differential equations in which the required functions depend only on z, and the first integrals are Hugoniot conditions. For these Hugoniot conditions, the determinacy theorem [3] is proved, which, in particular, implies that the values of $\boldsymbol{U}^1$ are uniquely determined from the specified parameters $\boldsymbol{U}^0$ and D.

In the present paper, the same approach is applied to a multicomponent medium in the case of an arbitrary number of components of the medium, i.e., for $N\geq 2$. For this, the first three equations of the obtained system of ordinary differential equations, i.e., Eqs. (3), are integrated over z and similar terms in them are cancelled. As a result, we obtain three groups of relations:

$$\sigma_i(u_i-D)=C_{1,i},$$

$$\sigma_i u_i(u_i-D)+(\gamma_i-1)c_{vi}^0\sigma_i T_i-\sigma_i(u-u_i)^2/2+I_i(z)=C_{2,i}, \qquad (5)$$

$$c_{vi}^0\sigma_i T_i(\gamma_i u_i-D)+\sigma_i u_i^2(u_i-D)/2-\sigma_i u_i(u-u_i)^2/2+\gamma_i c_{vi}^0\sigma_i T_i(u-u_i)/2+J_i(z)=C_{3,i}.$$

Here $i=1,2,\ldots,N$; $C_{1,i}$, $C_{2,i}$, and $C_{3,i}$ are arbitrary constants obtained by integrating system (3) over z, and

$$I_i(z)=\int_z^{+\infty}\alpha_i\sum_{j=1}^{N}\alpha_j a_{ji}[u_j(\xi)-u_i(\xi)]\,d\xi,$$

$$J_i(z)=\int_z^{+\infty}\alpha_i\sum_{j=1}^{N}\alpha_j\Big\{b_{ji}[P_j(\xi)-P_i(\xi)]+c_{ji}[T_j(\xi)-T_i(\xi)]+0.5a_{ji}[u_j^2(\xi)-u_i^2(\xi)]\Big\}\,d\xi.$$

Remark 3. It is assumed that, as $-\infty<z<+\infty$, all improper integrals converge.

Remark 4. The integrals $I_i(z)$ and $J_i(z)$ are obtained with allowance for the value of the derivative of the integral with a variable lower limit

$$\Big(\int_x^b f(\xi)\,d\xi\Big)'_x=-f(x).$$

We will further study the properties of only those traveling waves for which the parameters of all components take constant values as $z\to\pm\infty$:

$$\lim_{z\to+\infty}\boldsymbol{U}_i(z)=\boldsymbol{U}_i^0,\qquad \lim_{z\to-\infty}\boldsymbol{U}_i(z)=\boldsymbol{U}_i^1,\qquad 1\leq i\leq N \qquad (6)$$

(the coordinates of the vectors $\boldsymbol{U}_i$ are the quantities σ_i, u_i, T_i, and α_i).

Assuming that, for $z=\pm\infty$, multicomponent flows are in equilibrium in velocities, temperatures, and pressures:

$$u_i^0=u^0,\quad T_i^0=T^0,\quad P_i^0=P^0,\quad u_i^1=u^1,\quad T_i^1=T^1,\quad P_i^1=P^1 \qquad (7)$$

$(1 \leq i \leq N)$, we write the relations

$$\rho_i^0 = \frac{P^0}{(\gamma_i - 1)c_{vi}^0 T^0}, \qquad \sigma_i^0 = \alpha_i^0 \rho_i^0, \qquad \rho_i^1 = \frac{P^1}{(\gamma_i - 1)c_{vi}^0 T^1}, \qquad \sigma_i^1 = \alpha_i^1 \rho_i^1, \tag{8}$$

where $1 \leq i \leq N$. We will also assume that, for the traveling waves considered, the following equalities hold as $z \to \pm\infty$:

$$\sum_{i=1}^{N} \alpha_i^0 = 1, \qquad \sum_{i=1}^{N} \alpha_i^1 = 1. \tag{9}$$

Since the purpose of the present work is to determine the constant parameters of the multicomponent medium $\boldsymbol{U}_i^1$ for $z = -\infty$ [using assumptions (6)–(9) and from the traveling wave velocity D and the parameters of the multicomponent medium $\boldsymbol{U}_i^0$ specified for $z = +\infty$], it is necessary to obtain conditions similar to the Hugoniot conditions. However, concrete flows will not be constructed.

Remark 5. In [5] this model of a multicomponent medium is used to consider flow with one SW, on both sides of which the velocities, temperatures, and pressures are in equilibrium. In [5], it is proved that, in this case, the parameters of each component on the different sides of the SW are linked by the usual Hugoniot conditions [3, 4]. Then, in the general case where the thermodynamic parameters of the components are different and the SW velocity is identical for all components, equilibrium of the velocities behind the SW is impossible. Therefore, in the present paper, the number of strong flow discontinuities is arbitrary and equilibrium for velocities, temperatures, and pressures is assumed only in the limit as $z \to \pm\infty$ but not in the middle part of the flow.

In view of conditions (6) and (7) and the equalities

$$\lim_{z \to +\infty} I_i(z) = 0, \qquad \lim_{z \to +\infty} J_i(z) = 0,$$

which hold under the assumption of convergence Of the improper integrals, we pass to the limit as $z \to +\infty$ in relations (5). As a result, we obtain relations from which the constants $C_{1,i}$, $C_{2,i}$, and $C_{3,i}$ are uniquely determined using the values of D and $\boldsymbol{U}_i^0$:

$$\begin{gathered} C_{1,i} = \sigma_i^0(u^0 - D), \qquad C_{2,i} = \sigma_i^0 u^0 (u^0 - D) + (\gamma_i - 1)c_{vi}^0 \sigma_i^0 T^0, \\ C_{3,i} = c_{vi}^0 \sigma_i^0 T^0 (\gamma_i u^0 - D) + 0.5\sigma_i^0 (u^0)^2 (u^0 - D), \qquad 1 \leq i \leq N. \end{gathered} \tag{10}$$

In the derivation of relations (10), we took into account that

$$\lim_{z \to +\infty} u(z) = \sum_{i=1}^{N} \sigma_i^0 u^0 \Big/ \sum_{i=1}^{N} \sigma_i^0 = u^0;$$

therefore, $\lim\limits_{z \to +\infty} (u - u_i) = 0$.

We introduce the notation

$$I_i^1 = \lim_{z \to -\infty} I_i(z) = \int_{-\infty}^{+\infty} \alpha_i(z) \sum_{j=1}^{N} \alpha_j(z) a_{ji} [u_j(z) - u_i(z)]\, dz,$$

$$J_i^1 = \lim_{z \to -\infty} J_i(z) = \int_{-\infty}^{+\infty} \alpha_i(z) \sum_{j=1}^{N} \alpha_j(z) \Big\{ b_{ji}[P_j(z) - P_i(z)] + c_{ji}[T_j(z) - T_i(z)] + 0.5a_{ji}[u_j^2(z) - u_i^2(z)] \Big\}\, dz,$$

and, taking into count that $\lim\limits_{z \to -\infty} (u - u_i) = 0$, we pass to the limit as $z \to -\infty$ in relations (5). As a result, for the unknown quantities σ_i^1, u^1, T^1, I_i^1, and J_i^1 $(i = 1, 2, \ldots, N)$, we obtain the functional equations

$$\sigma_i^1 (u^1 - D) = C_{1,i}; \tag{11}$$

$$\sigma_i^1 u^1 (u^1 - D) + (\gamma_i - 1)c_{vi}^0 \sigma_i^1 T^1 + I_i^1 = C_{2,i}; \tag{12}$$

$$c_{vi}^0 \sigma_i^1 T^1 (\gamma_i u^1 - D) + \sigma_i^1 (u^1)^2 (u^1 - D)/2 + J_i^1 = C_{3,i}. \tag{13}$$

From Eqs. (11), we have

$$\sigma_i^1 = \frac{C_{1,i}}{u^1 - D}, \qquad 1 \le i \le N, \tag{14}$$

which allows the quantity σ_i^1 to be eliminated from Eqs. (12) and (13):

$$C_{1,i}u^1 + (\gamma_i - 1)c_{vi}^0 C_{1,i}\,\frac{T^1}{u^1 - D} + I_i^1 = C_{2,i}; \tag{15}$$

$$c_{vi}^0 C_{1,i}\,\frac{T^1}{u^1 - D}\,(\gamma_i u^1 - D) + \frac{1}{2}\,C_{1,i}(u^1)^2 + J_i^1 = C_{3,i}, \tag{16}$$

where $i = 1, 2, \ldots, N$.

Equations (15) are summed over $i = 1, 2, \ldots, N$:

$$\Big(\sum_{i=1}^{N} C_{1,i}\Big)u^1 + \Big(\sum_{i=1}^{N}(\gamma_i - 1)c_{vi}^0 C_{1,i}\Big)\frac{T^1}{u^1 - D} + \sum_{i=1}^{N} I_i^1 = \sum_{i=1}^{N} C_{2,i}.$$

Similar summation of Eqs. (16) over i leads to the equality

$$\Big(\sum_{i=1}^{N} c_{vi}^0 C_{1,i}\gamma_i\Big)\frac{T^1}{u^1 - D}\,u^1 - \Big(\sum_{i=1}^{N} c_{vi}^0 C_{1,i}\Big)\frac{T^1}{u^1 - D}\,D + \frac{1}{2}\Big(\sum_{i=1}^{N} C_{1,i}\Big)(u^1)^2 + \sum_{i=1}^{N} J_i^1 = \sum_{i=1}^{N} C_{3,i}.$$

Theorem 1. *The following equalities are valid*:

$$\sum_{i=1}^{N} I_i^1 = \sum_{i=1}^{N} J_i^1 = 0.$$

Proof. The above sums are integrals of linear expressions of the type

$$\sum_{i=1}^{N}\sum_{j=1}^{N}\Big\{d_{i,j}[g_j(z) - g_i(z)]\Big\}, \qquad d_{i,j} = d_{j,i}.$$

If $k = l$, then $g_k(z) - g_l(z) \equiv 0$. In another case for any pair of different integer numbers (k, l), where $k \ne l$, each of the sums contain two such terms:

$$d_{k,l}[g_k(z) - g_l(z)], \qquad d_{l,k}[g_l(z) - g_k(z)].$$

In view of the equality $d_{k,l} = d_{l,k}$, the sum of these terms is identically equal to zero. Since it is assumed that the improper integrals converge, the summation and integration operations can be interchanged. As a result of this rearrangement, the subintegral functions become equal to zero and, hence, the integrals also become equal to zero. The theorem is proved.

Remark 6. The equality to zero of the sums of the integrals I_i^1 and J_i^1 $(1 \le i \le N)$ follows from the conservation laws for momentum and energy (including the case of interaction of different components) underlying the model of multicomponent media considered [1].

Thus, the summation results in the following equations for the two required quantities u^1 and T^1:

$$C_1 u^1 + C_4\,\frac{T^1}{u^1 - D} = C_2; \tag{17}$$

$$(C_4 + C_5)\,\frac{T^1}{u^1 - D}\,u^1 - C_5\,\frac{T^1}{u^1 - D}\,D + \frac{1}{2}\,C_1(u^1)^2 = C_3. \tag{18}$$

Here

$$C_1 = \sum_{i=1}^{N} C_{1,i}, \qquad C_2 = \sum_{i=1}^{N} C_{2,i}, \qquad C_3 = \sum_{i=1}^{N} C_{3,i},$$

$$C_4 = \sum_{i=1}^{N}(\gamma_i - 1)c_{vi}^0 C_{1,i}, \qquad C_5 = \sum_{i=1}^{N} c_{vi}^0 C_{1,i}, \qquad \sum_{i=1}^{N}\gamma_i c_{vi}^0 C_{1,i} = C_4 + C_5.$$

Expressing the quantity $T^1/(u^1 - D)$ from Eq. (17)

$$\frac{T^1}{u^1 - D} = \frac{C_2 - C_1 u^1}{C_4}, \tag{19}$$

for the velocity u^1 we obtain the quadratic equation

$$(C_4 + C_5)\,\frac{C_2 - C_1 u^1}{C_4}\,u^1 - C_5\,\frac{C_2 - C_1 u^1}{C_4}\,D + \frac{1}{2}\,C_1 (u^1)^2 = C_3,$$

which can be written in the traditional form

$$A(u^1)^2 + Bu^1 + E = 0, \tag{20}$$

where

$$A = \frac{1}{2}\,C_1 - \frac{C_1(C_4 + C_5)}{C_4} = \frac{C_1[C_4 - 2(C_4 + C_5)]}{2C_4} = -\frac{C_1(C_4 + 2C_5)}{2C_4},$$

$$B = \frac{C_2(C_4 + C_5) + C_1 C_5 D}{C_4}, \qquad E = -\frac{C_2 C_5 D}{C_4} - C_3.$$

One of the roots of Eq. (20) is the parameter u^0. Indeed, if in relations (10), which do not contain the integrals $I_i(z)$ and $J_i(z)$, we make the same actions as in relations (11)–(13) (elimination of σ_i^0, summation of two groups of equations over i, and elimination of T^0), we obtain

$$A(u^0)^2 + Bu^0 + E = 0.$$

According to the Vieta theorem, the roots of Eq. (20) satisfy the equality

$$u^1 + u^0 = -B/A,$$

Hence,

$$u^1 = -B/A - u^0. \tag{21}$$

Determining the value of u^1 by formula (21), from relations (14) we find the values of σ_i^1 $(i = 1, 2, \ldots, N)$. Then, from (19) we obtain $T^1 = (u^1 - D)(C_2 - C_1 u^1)/C_4$. After that, we find α_i^1 $(i = 1, 2, \ldots, N)$ as follows. For $z \to -\infty$, there is pressure equilibrium:

$$(\gamma_1 - 1)c_{v1}^0 \rho_1^1 T^1 = (\gamma_j - 1)c_{vj}^0 \rho_j^1 T^1, \qquad j = 2, \ldots, N.$$

Therefore,

$$\frac{\rho_1^1}{\rho_j^1} = \frac{(\gamma_j - 1)c_{vj}^0}{(\gamma_1 - 1)c_{v1}^0}.$$

Because $\sigma_i^1 = \alpha_i^1 \rho_i^1$ $(i = 1, 2, \ldots, N)$, it follows that $\alpha_i^1 = \sigma_i^1/\rho_i^1$, and, hence,

$$\frac{\alpha_j^1}{\alpha_1^1} = \frac{\sigma_j^1 \rho_1^1}{\rho_j^1 \sigma_1^1} = \frac{\sigma_j^1(\gamma_j - 1)c_{vj}^0}{\sigma_1^1(\gamma_1 - 1)c_{v1}^0}, \qquad j = 2, \ldots, N, \tag{22}$$

where the fractions on the right side are already known. We denote these fractions as

$$k_j = \frac{\sigma_j^1(\gamma_j - 1)c_{vj}^0}{\sigma_1^1(\gamma_1 - 1)c_{v1}^0} > 0, \qquad j = 2, \ldots, N$$

and write equality (22) as

$$\alpha_j^1 = k_j \alpha_1^1, \qquad j = 2, \ldots, N. \tag{23}$$

According to expressions (9), we have

$$\alpha_1^1(1 + k_2 + \ldots + k_N) = 1.$$

From this, we determine

$$\alpha_1^1 = 1/(1 + k_2 + \ldots + k_N),$$

and then, by formulas (23), we find the remaining values α_j^1 $(j = 2, \ldots, N)$.

Using relation (14) for σ_i^1 and taking into account the constants $C_{1,i}$ (10) given by equalities (10), from the condition of flow pressure equilibrium for $z \to \pm\infty$, we obtain

$$k_j = \alpha_j^0/\alpha_1^0, \qquad j = 2, \ldots, N. \tag{24}$$

Indeed,

$$k_j = \frac{\sigma_j^1(\gamma_j - 1)c_{vj}^0}{\sigma_1^1(\gamma_1 - 1)c_{v1}^0} = \frac{C_{1,j}(\gamma_j - 1)c_{vj}^0}{C_{1,1}(\gamma_1 - 1)c_{v1}^0} = \frac{\sigma_j^0(\gamma_j - 1)c_{vj}^0}{\sigma_1^0(\gamma_1 - 1)c_{v1}^0} = \frac{\alpha_j^0\rho_j^0(\gamma_j - 1)c_{vj}^0 T^0}{\alpha_1^0\rho_1^0(\gamma_1 - 1)c_{v1}^0 T^0} = \frac{\alpha_j^0 P_j^0}{\alpha_1^0 P_1^0} = \frac{\alpha_j^0}{\alpha_1^0}.$$

From formulas (23) and (24), it follows that

$$1 + \sum_{j=2}^{N} k_j = 1 + \sum_{j=2}^{N} \frac{\alpha_j^1}{\alpha_1^1} = 1 + \sum_{j=2}^{N} \frac{\alpha_j^0}{\alpha_1^0}.$$

Therefore, in view of equality (9), we have

$$\alpha_1^1 = \alpha_1^0.$$

Then, from relations (23) and (24), we obtain

$$\alpha_j^1 = \alpha_j^0, \qquad j = 2, \ldots, N.$$

After that, from relations (12) and (13), we find the remaining required quantities

$$I_i^1 = C_{2,i} - \sigma_i^1 u^1(u^1 - D) - (\gamma_i - 1)c_{vi}^0\sigma_i^1 T^1,$$

$$J_i^1 = C_{3,i} - c_{vi}^0\sigma_i^1 T^1(\gamma_i u^1 - D) - \sigma_i^1(u^1)^2(u^1 - D)/2, \qquad 1 \le i \le N.$$

Thus, in the particular case where, as $z \to \pm\infty$, the medium with N components is in homogeneous states at equilibrium velocities, temperatures, and pressures

$$\lim_{z\to+\infty} \boldsymbol{U}_i(z) = \boldsymbol{U}_i^0, \qquad \lim_{z\to-\infty} \boldsymbol{U}_i(z) = \boldsymbol{U}_i^1,$$

the following theorem similar to the determinacy theorem [3] is proved.

Theorem 2. *If equalities (6)–(9) are valid, the parameter $\boldsymbol{U}_i^1$ and the integrals I_i^1 and J_i^1 are uniquely determined from specified values of $\boldsymbol{U}_i^0$ and the traveling wave velocity D; in this case, $\alpha_i^1 = \alpha_i^0$ $(1 \le i \le N)$.*

In the case of a one-component medium ($N = 1$), i.e., in the case

$$c_{vi}^0 = c_v^0, \qquad \gamma_i = \gamma, \tag{25}$$

formula (21) for $u^0 = 0$ becomes formula [4]

$$u^1 = \frac{2}{\gamma + 1}\Big(D - \frac{(c^0)^2}{D}\Big). \tag{26}$$

In this case, the following constants are used:

$$C_1 = -D\rho^0, \qquad C_2 = (-D\rho^0)(\gamma - 1)c_v^0 T^0/(-D),$$

$$C_3 = (-D\rho^0)c_v^0 T^0, \qquad C_4 = (-D\rho^0)(\gamma - 1)c_v^0, \qquad C_5 = (-D\rho^0)c_v,$$

$$A = -(-D\rho^0)\frac{\gamma + 1}{2(\gamma - 1)}, \qquad B = \frac{-D\rho^0}{\gamma - 1}\Big(D - \frac{(c^0)^2}{D}\Big).$$

The transformations resulting in formula (26) are not given here due to their length.

The proposed technique can be used in the case of other equations of state for the components. Similarly, it is possible to obtain equations corresponding to Eqs. (17) and (18) which are functional equations for the required quantities u^1 and T^1 for specified D and $\boldsymbol{U}_i^0$. The form of these functional equations is determined by the form of the equations of state for the components. This approach can also be applied to other systems of equations with partial derivatives, whose solutions describe flows in media in which there is the corresponding interaction of various components, for example, to the equations describing flows in a multitemperature plasma [6, 7].

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